Although calculations of this type do not predict the proper behavior for the ω-β characteristic near stopbands resulting from a periodic perturbation, they do predict the occurrence and width of such stop bands.

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Ice as a Bending Medium for Waveguide and Tubing*

Bending waveguide and metal tubing is very often a difficult and time-consuming task. Low melting temperature alloys are at times difficult to remove from waveguide and tubing. The piece to be bent may be filled with water which is then frozen by dry ice, liquid nitrogen, or by a deep freeze. In some applications where the piece to be bent is integral with a larger system, a block of dry ice may be held against it to freeze only the portion of water around the section to be bent. The use of these low temperatures causes not only the water to freeze into quite small crystals (which act like a sand packing), but also prevents the ice from melting because of the pressure of bending.

Several tests were performed on thin walled aluminum tubing and P-band brass waveguide. It was found that in comparison to low melting alloys the bends were identical within the statistical variation of samples. The time required for the operation was considerably shorter.

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On Higher-Order Hybrid Modes of Dielectric Cylinders*

In the course of investigations into the properties of various surface wave structures,[1] it became necessary to investigate hybrid modes on dielectric cylinders for modes of order n, where $n>1$. The case $n=1$ has received extensive treatment in the literature [1]–[6].

The radial dependence of the axial fields is as $J_n[p(\rho/a)]$ inside the dielectric cylinder and $K_n[q(\rho/a)]$ outside, where ρ is the radial cylindrical coordinate, a is the radius of the cylinder, p and q are radial eigenvalues, and n is the rank of the mode.

The requirement of continuity of the fields at the boundary leads, in the usual manner, to the characteristic equation involving Bessel functions and their derivatives. This was first given by Schelkunoff [4]. The derivatives of Bessel functions may be eliminated from this equation by the use of identities such as given by Watson [8], to yield the simple form

$$(J^+ + K^+)(\epsilon J^- - K^-) + (J^- - K^-)(\epsilon J^+ + K^+) = 0, \quad (1)$$

where

$$J^- = \frac{J_{n-1}(p)}{pJ_n(p)}, \quad J^+ = \frac{J_{n+1}(p)}{pJ_n(p)};$$
$$K^- = \frac{K_{n-1}(q)}{qK_n(q)}, \quad K^+ = \frac{K_{n+1}(q)}{qK_n(q)};$$

and ϵ is permittivity of dielectric cylinder relative to surrounding medium.

The cutoff values of the parameter p are of great interest; they may be obtained by letting $q\to 0$ in the characteristic equation. To keep the terms finite requires that the equation be multiplied by an appropriate power of q before the limit is taken. If it is assumed that J^- is finite at cutoff, it is sufficient to multiply the equation by q^2 to obtain a solution for the cutoff values of p; this was given by Schelkunoff [4]. However, if this assumption is not made, an additional solution may be determined. This will be outlined below.

Multiplying the characteristic equation by $[qpJ_n(p)]^2$ gives

$$(q^2J_{n+1} + q^2K^+pJ_n)(\epsilon J_{n-1} - pJ_nK^-) + (J_{n-1} - pJ_nK^-)(\epsilon q^2J_{n+1} + q^2K^+pJ_n) = 0. \quad (2)$$

Taking the limit as $q\to 0$ and noting that

$$K^- \to \frac{1}{2(n-1)}$$

and $q^2K^+\to 2n$ one obtains

$$2n\, pJ_n\left((\epsilon+1)J_{n-1} - \frac{pJ_n}{n-1}\right) = 0. \quad (3)$$

The solutions are, for $n>1$,

$$\frac{J_{n-1}(p)}{pJ_n(p)} = J^- = \frac{1}{(n-1)(\epsilon+1)}, \quad (4)$$
$$J_n(p) = 0, \qquad p \neq 0. \quad (5)$$

Eq. 4 is given by Schelkunoff [4]. The very significant exclusion of the $p=0$ solution of (5) as a cutoff condition is based on the fact that for $q\to 0$ and $p\to 0$, (1) becomes, since

$$J^- \to \frac{2n}{p^2}, \quad J^+ \to \frac{1}{2(n+1)},$$

$$\left(\frac{1}{2(n+1)}+\frac{2n}{q^2}\right)\left(\frac{2n\epsilon}{p^2}-\frac{1}{2(n-1)}\right) + \left(\frac{2n}{p^2}-\frac{1}{2(n-1)}\right)\left(\frac{\epsilon}{2(n+1)}+\frac{2n}{q^2}\right) = 0. \quad (6)$$

When the finite terms are neglected in comparison with the infinite terms, it is seen that this is not satisfied at $q=0$, $p=0$ for any $n>1$. However, the $p=q=0$ solution, *i.e.*, the condition for "no cutoff," is valid for $n=1$ [1].

The asymptotes for the p–q curves are of interest. For $q\to\infty$ the characteristic equation becomes simply $2\epsilon J^-J^+=0$, with solutions at $J_{n-1}(p)=0$ and $J_{n+1}(p)=0$. It will be seen that the first of these is associated with the modes satisfying the first or Schelkunoff cutoff condition, the second with the alternate cutoff condition given here in (5).

Because of the oscillatory character of $J_n(p)$, the characteristic equation is satisfied by an infinite set of values of p for any given q, in particular also for $q=0$. These sets of p's span an infinite set of modes which may propagate along the dielectric rod. It is now seen that the existence of the alternate cutoff condition indicates the existence of an infinite set of modes that interlace the modes that satisfy the cutoff condition of (4). This and other salient characteristics of the doubly infinite set of modes are presented qualitatively in Fig. 1, with the $n=1$ case treated by Beam [1] included for comparison in Fig. 2. The curve shapes are based upon the detailed numerical solution of (2) obtained with an IBM 650 computer for $n=2$, 6 for a wide range of ϵ.

The significance of Fig. 1 may be summarized as follows.

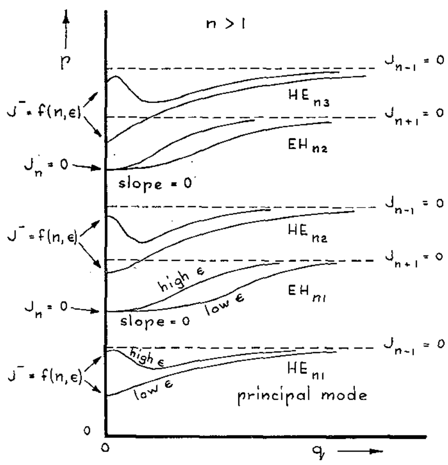

Fig. 1—Loci of solutions of the characteristic equation (1) for $n>1$.

$$f(n,\epsilon) = \frac{1}{(n-1)(\epsilon+1)}.$$

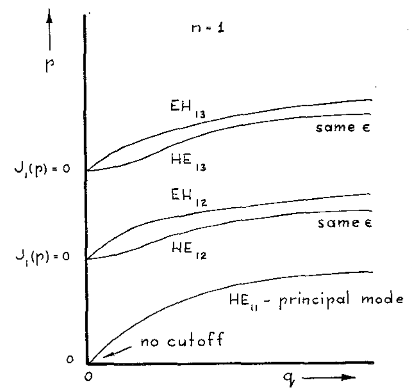

Fig. 2—Curves of p and q for $n=1$.

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[1] Report in preparation.